Problem-2: Applied Mathematics One(Math 1021)

By:- Dejen Ketema *

Arba Minch University

Department of Mathematics

December 20, 2018

Focus Group:*UG1 Environmental Health*

"**Attention!** Every student must be exercise and understand this practical problems and the content that we discussed in chapter two before **December 25** as far as possible."

*dejen.ketema@amu.edu.et **Note**: If you have any comment and suggestion please contacts me.

1. Given that $\begin{vmatrix} r & s & t \\ u & v & w \\ x & y & z \end{vmatrix} = 4$, evaluate the determinate.

   (a) $\begin{vmatrix} -r & -s & -t \\ 3u & 3v & 3w \\ 2x & 2y & 2z \end{vmatrix}$

   (b) $\begin{vmatrix} r & s & t \\ 8u & 8v & 8w \\ x-8r & y-8s & z-8t \end{vmatrix}$

   (c) $det(A^{-1})$

   (d) $det(A^2)$

   a) $\begin{vmatrix} -r & -s & -t \\ 3u & 3v & 3w \\ 2x & 2y & 2z \end{vmatrix} = -\begin{vmatrix} r & s & t \\ 3u & 3v & 3w \\ 2x & 2y & 2z \end{vmatrix} = (-3)\begin{vmatrix} r & s & t \\ u & v & w \\ 2x & 2y & 2z \end{vmatrix} = (-6)\begin{vmatrix} r & s & t \\ u & v & w \\ x & y & z \end{vmatrix} = (-6)4 = -24$

   b) $\begin{vmatrix} r & s & t \\ 8u & 8v & 8w \\ x-8r & y-8s & z-8t \end{vmatrix} = 8\begin{vmatrix} r & s & t \\ u & v & w \\ x-8r & y-8s & z-8t \end{vmatrix} = 8\begin{vmatrix} r & s & t \\ u & v & w \\ x & y & z \end{vmatrix} = 8(4) = 32$

   c) $det(A^{-1}) = 1/4$

   d) $det(A^2) = 4^2 = 8$

2. Manufacturing: Production Scheduling Ace Novelty wishes to produce three types of souvenirs: types A, B, and C. To manufacture a type-A souvenir requires 2 minutes on machine I, 1 minute on machine II, and 2 minutes on machine III. A type-B souvenir requires 1 minute on machine I, 3 minutes on machine II, and 1 minute on machine III. A type-C souvenir requires 1 minute on machine I and 2 minutes each on machines II and III. There are 3 hours available on machine I, 5 hours available on machine II, and 4 hours available on machine III for processing the order. How many souvenirs of each type should Ace Novelty make in order to use all of the available time? Formulate but do not solve the problem.

3. Let's begin by considering how the monthly output data of a manufacturer may be organized. The Acrosonic Company manufactures four different loudspeaker systems at three separate locations. The company's May output is described in Table 1.

	Model A	Model B	Model C	Model D
Location I	320	280	460	280
Location II	480	360	580	0
Location III	540	420	200	880

Organizing Production data Consider the matrix

$$P = \begin{bmatrix} 320 & 280 & 460 & 280 \\ 480 & 360 & 580 & 0 \\ 540 & 420 & 200 & 880 \end{bmatrix}$$

representing the output of loudspeaker systems of the Acrosonic Company discussed earlier (see Table 1).

(a) What is the size of the matrix P?

(b) Find p_{24} (the entry in row 2 and column 4 of the matrix P), and give an interpretation of this number.

(c) Find the sum of the entries that make up row 1 of P, and interpret the result.

(d) Find the sum of the entries that make up column 4 of P, and interpret the result.

4. Organizing Production data The total output of Acrosonic for June is shown in Table 2

	Model A	Model B	Model C	Model D
Location I	210	180	330	180
Location II	400	300	450	40
Location III	420	280	180	740

The output for May was given earlier, in Table above 1. Find the total output of the company for May and June.

5. AMU shop store has two locations 'A' and 'B', and their sales of soaps are given by type (in rows) and quarters {1/4 of a year} (in columns) as shown below.

$$A = \begin{bmatrix} 27 & 7 & 16 & 6 \\ 25 & 15 & 10 & 5 \\ 2 & 3 & 20 & 25 \end{bmatrix} \text{ and } B = \begin{bmatrix} 20 & 7 & 1 & 4 \\ 21 & 15 & 6 & 3 \\ 0 & 4 & 5 & 20 \end{bmatrix}$$

where the rows represent sale of Diva, Medical and Popular and the columns represent the quarter number 1, 2, 3, 4.

(a) What are the total sales of the two locations by type and quarter?

(b) How many more sops did store A sell than store B of each brand in each quarter?

(c) Find the per quarter sales of store A if following are the prices of each soap, where Diva = \$33.25, Medical = \$40.19 and Popular=\$25.03

a)

$$[C] = [A] + [B] = \begin{bmatrix} 45 & 25 & 7 & 2 \\ 8 & 16 & 30 & 46 \\ 10 & 17 & 14 & 47 \end{bmatrix}$$

So if one wants to know the total number of Diva soap sold in quarter 4 in the two locations, we would look at Row 3 - Column 4 to give $C_{34} = 47$

b) $[D] = [A] - [B] = \begin{bmatrix} 5 & 15 & -1 & 2 \\ 2 & 4 & 0 & 4 \\ 2 & 15 & 0 & 7 \end{bmatrix}$

So if you want to know how many more Diva soap were sold in quarter 4 in Store A than Store B, $d_{34} = 7$. Note that $d_{13} = -1$ implying that store A sold 1 less Medical soap than Store B in quarter 3.

c) The answer is given by multiplying the price matrix by the quantity sales of store A. The price matrix is $\begin{bmatrix} 33.25 & 40.19 & 25.03 \end{bmatrix}$, then the per quarter sales of store A would be given by $[C] = \begin{bmatrix} 33.25 & 40.19 & 25.03 \end{bmatrix} \begin{bmatrix} 27 & 7 & 16 & 6 \\ 25 & 15 & 10 & 5 \\ 2 & 3 & 20 & 25 \end{bmatrix}$

So each quarter sales of store A in dollars are given by the four columns of the row vector. $[C] = \begin{bmatrix} 1182.38 & 1467.38 & 877.81 & 1747.06 \end{bmatrix}$

6. Food orders are taken from section G_1 and G_2 Environmental Health class for a takeout. The order is tabulated below.

Food order: $\begin{matrix} \textit{Electrical } G_1 \\ \textit{Electrical } G_2 \end{matrix} \begin{bmatrix} \textit{Cream cake} & \textit{Pizza} & \textit{Fish Gulash} \\ 25 & 35 & 25 \\ 21 & 20 & 21 \end{bmatrix}$.

However they have a choice of buying this food from three different restaurants. Their prices for the three food items are tabulated below

Price Matrix: $\begin{matrix} \textit{Cream cake} \\ \textit{Pizza} \\ \textit{Fish Gulash} \end{matrix} \begin{bmatrix} \textit{Tourist} & \textit{Paradise} & \textit{JBM} \\ 2.42 & 3.38 & 2.46 \\ 0.93 & 0.90 & 0.89 \\ 0.95 & 1.03 & 1.13 \end{bmatrix}$

Show how much each class will pay for their order at each restaurant. Which restaurant would be more economical to order from for each class?

Solution: By using the product between food order and price matrix. The cost in dollars is $116.80, 116.75, 120.90$ for the G_1A Department at three fast food joints. So Paradise is the cheapest for the G_1A class.
The cost in dollars is $89.37, 89.61, 93.19$ for the G_1B class at three fast food joints. Tourist is the cheapest for the G_1B class.

7. The upward velocity of a rocket is given at three different times on the following table

Time, t	velocity, v
s	m/s
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as

$$v(t) = at^2 + bt + c, \qquad 5 \leq t \leq 12$$

(a) Set up the equations in matrix form to find the coefficients a, b, c of the velocity profile.

(b) Solve the coefficients a, b, c by using

i. Cramer's rule

ii. Gauss elimination and find the velocity at $t = 6, 7.5, 9, 11$ seconds.

iii. Inverse method

The polynomial is going through three data points (t_1, v_1), (t_2, v_2) and (t_3, v_3) where from the above table

$$\begin{aligned} t_1 &= 5 & v_1 &= 106. \\ t_2 &= 8 & v_2 &= 177.2 \\ t_3 &= 12 & v_3 &= 2279.2 \end{aligned}$$

Requiring that $v(t) = at^2 + bt + c$ passes through the three data points gives

$$v(t_1) = at_1^2 + bt_1 + c$$
$$v(t_2) = at_2^2 + bt_2 + c$$
$$v(t_3) = at_3^2 + bt_3 + c$$

Substituting the data (t_1, v_1), $(t_2, v_2), (t_3, v_3)$ gives

$$25a + 5b + c = 106.8$$
$$64a + 8b + c = 177.2$$
$$144a + 12b + c = 279.2$$

This set of equations can be rewritten in the matrix form as

$$\begin{bmatrix} 25a + 5b + c \\ 64a + 8b + c \\ 144a + 12b + c \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

The above equation can be written as a linear combination as follows

$$a \begin{bmatrix} 25 \\ 64 \\ 144 \end{bmatrix} + b \begin{bmatrix} 5 \\ 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

and further using matrix multiplications gives

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0.0476 & -0.0833 & 0.0357 \\ -0.9524 & 1.4167 & -0.4643 \\ 4.5714 & -5.0000 & 1.4286 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.2905 \\ 19.6905 \\ 1.0857 \end{bmatrix}$$

So

$$v(t) = 0.2905t^2 + 19.6905t + 1.0857, \quad 5 \leq t \leq 12$$

$$v(6) = 129.7m/s$$
$$v(7.5) = 165.1m/s$$
$$v(9) = 201.8m/s$$
$$v(11) = 252.8m/s$$

8. Suppose an economy consists of the Coal, Electric (power), and Steel sectors, and the output of each sector is distributed among the various sectors as shown in the Table below, where the entries in a column represent the fractional parts of a sector's total output.

Distribution of output form:			
Coal	Electric	steel	purchased by
0	0.4	0.6	coal
0.6	0.1	0.2	electric
0.4	0.5	0,2	steel

The second column of the Table, for instance, says that the total output of the Electric sector is divided as follows: 40% to Coal, 50% to Steel, and the remaining 10% to Electric. (Electric treats this 10% as an expense it incurs in order to operate its business.) Since all output must be taken into account, the decimal fractions in each column must sum to 1.

Denote the prices (i.e., dollar values) of the total annual outputs of the Coal, Electric, and Steel sectors by pC, pE, and pS, respectively. If possible, find equilibrium prices that make each sector's income match its expenditures.

Solution: We have

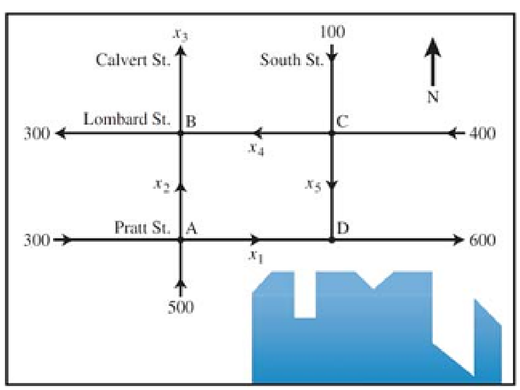
$$pC = .4pE + .6pS$$
$$pE = .6pC + .1pE + .2pS$$
$$pS = .4pC + .5pE + .2pS$$

hence

$$pC - .4pE - .6pS = 0$$
$$-.6pC - 0.9pE - .2pS = 0$$
$$-.4pC - .5pE + .8pS = 0$$

The general solution is pC=.94pS, pE= .85pS, and pS is free. The equilibrium price vector for the economy has the form $\begin{bmatrix} pC \\ pE \\ pS \end{bmatrix} = \begin{bmatrix} .94pS \\ .85pS \\ pS \end{bmatrix} = pS \begin{bmatrix} 94 \\ .85 \\ 1 \end{bmatrix}$ Any (nonnegative) choice for pS results in a choice of equilibrium prices. For instance, if we take pS to be 100 (or \$100 million), then pC=94 and pE= 85. The incomes and expenditures of each sector will be equal if the output of Coal is priced at \$94 million, that of Electric at \$85 million, and that of Steel at \$100 million

9. The network in the Figure below shows the traffic flow (in vehicles per hour) over several one-way streets in downtown Adiss Abeba during a typical early afternoon. Determine the general flow pattern for the network.



Solution: Write equations that describe the flow, and then find the general solution of the system. Label the street intersections (junctions) and the unknown flows in the branches, as shown in the Figure above. At each intersection, set the flow in equal to the flow out.

Intersection	Flow in	Flow out
A	300+500=$x_1 + x_2$	
B	$x_2 + x_4$=300+x_3	
C	100+400=$x_4 + x_5$	
D	$x_1 + x_5$=600	

Also, the total flow into the network (500 + 300 + 100 + 400) equals the total flow out of the network $(300 + x_3 + 600)$, which simplifies to $x_3 = 400$. Combine this equation with a rearrangement of the first four equations to obtain the following system of equations:

$$x_1 + x_2 = 800$$
$$x_2 - x_3 + x_4 = 300$$
$$x_4 + x_5 = 500$$
$$x_1 + x_5 = 00$$
$$x_3 = 400$$

. The general flow pattern for the network is described by

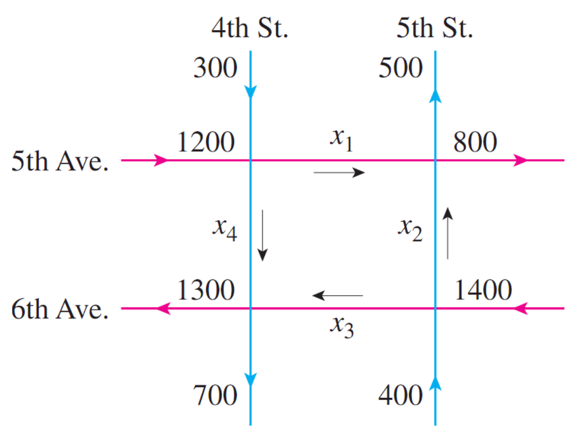
$$x_1 = 600 - x_5$$
$$x_2 = 200 - x_5$$
$$x_3 = 400$$
$$x_4 = 500 - x_5$$
$$x_5 \text{ is free}$$

A negative flow in a network branch corresponds to flow in the direction opposite to that shown on the model. Since the streets in this problem are one-way, none of the variables here can be negative. This fact leads to certain limitations on the possible values of the variables. For instance, $x_5 \leq 500$ because x_4 cannot be negative.

10. Find the eigenvalue and corresponding eigenvectors of

$$A = \begin{bmatrix} 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ -2 & -2 & 3 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

11. Traffic Control Figure blow shows the flow of downtown traffic in a certain city during the rush hours on a typical week- day. The arrows indicate the direction of traffic flow on each one-way road, and the average number of vehicles per hour entering and leaving each intersection appears beside each road. 5th Avenue and 6th Avenue can each handle up to 2000 vehicles per hour without causing congestion, whereas the maximum capacity of both 4th Street and 5th Street is 1000 vehicles per hour. The flow of traffic is controlled by traffic lights installed at each of the four intersections.



(a) Write a general expression involving the rates of flow x_1, x_2, x_3, x_4 and suggest two possible flow patterns that will ensure no traffic congestion.

(b) Suppose the part of 4th Street between 5th Avenue and 6th Avenue is to be resurfaced and that traffic flow between the two junctions must therefore be reduced to at most 300 vehicles per hour. Find two possible flow patterns that will result in a smooth flow of traffic.

Solution:

(a) To avoid congestion, all traffic entering an intersection must also leave that intersection. Applying this condition to each of the four intersections in a clockwise direction beginning with the 5th Avenue and 4th Street intersection, we obtain the following equations:

$$\begin{aligned}1500 &= x_1 + x_4\\1300 &= x_1 + x_2\\1800 &= x_2 + x_3\\2000 &= x_3 + x_4\end{aligned}$$

This system of four linear equations in the four variables x_1, x_2, x_3, x_4 may be rewritten in the more standard form

$$\begin{aligned}x_1 \qquad\qquad + x_4 &= 1500\\x_1 + x_2 \qquad\qquad &= 1300\\x_2 + x_3 \qquad &= 1800\\x_3 + x_4 &= 2000\end{aligned}$$

Using the Gauss–Jordan elimination method to solve the system, we obtain

$$\left[\begin{array}{cccc|c}1&0&0&1&1500\\1&1&0&0&1300\\0&1&1&0&1800\\0&0&1&1&2000\end{array}\right]\xrightarrow{R_2-R_1}\left[\begin{array}{cccc|c}1&0&0&1&1500\\0&1&0&-1&-200\\0&1&1&0&1800\\0&0&1&1&2000\end{array}\right]$$

$$\xrightarrow{R_3-R_2}\left[\begin{array}{cccc|c}1&0&0&1&1500\\0&1&0&-1&-200\\0&0&1&1&2000\\0&0&1&1&2000\end{array}\right]$$

$$\xrightarrow{R_4-R_3}\left[\begin{array}{cccc|c}1&0&0&1&1500\\0&1&0&-1&-200\\0&0&1&1&2000\\0&0&0&0&0\end{array}\right]$$

The last augmented matrix is in row-reduced form and is equivalent to a system of three linear equations in the four variables x_1, x_2, x_3, x_4. Thus, we may express three of the variables—say, x_1, x_2, x_3—in terms of x_4. Setting $x_4 = t$ (t a parameter), we may write the infinitely many solutions of the system as

$$\begin{aligned}x_1 &= 1500 - t\\x_2 &= -200 + t\\x_3 &= 2000 - t\\x_4 &= t\end{aligned}$$

Observe that for a meaningful solution we must have $200 \le t \le 1000$ since x_1, x_2, x_3, and x_4 must all be nonnegative and the maximum capacity of a street is 1000. For example, picking $t = 300$ gives the flow pattern

$$x_1 = 1200 \qquad x_2 = 100 \qquad x_3 = 1700 \qquad x_4 = 300$$

Selecting $t = 500$ gives the flow pattern

$$x_1 = 1000 \qquad x_2 = 300 \qquad x_3 = 1500 \qquad x_4 = 500$$

b. In this case, x_4 must not exceed 300. Again, using the results of part (a), we find, upon setting $x_4 = t = 300$, the flow pattern

$$x_1 = 1200 \qquad x_2 = 100 \qquad x_3 = 1700 \qquad x_4 = 300$$

obtained earlier. Picking $t = 250$ gives the flow pattern

$$x_1 = 1250 \qquad x_2 = 50 \qquad x_3 = 1750 \qquad x_4 = 250$$

12. The management of Acrosonic has decided to increase its July production of loudspeaker systems by 10% (over its June output). Find a matrix giving the targeted production for July.

Solution: The required matrix is given by

$$(1.1)B = 1.1\begin{bmatrix} 210 & 180 & 330 & 180 \\ 400 & 300 & 450 & 40 \\ 420 & 280 & 180 & 740 \end{bmatrix} = \begin{bmatrix} 231 & 198 & 363 & 198 \\ 440 & 330 & 495 & 44 \\ 462 & 308 & 198 & 814 \end{bmatrix}$$

and is interpreted in the usual manner.

13. **Stock Transactions :** Judy's stock holdings are given by the matrix

$$\begin{bmatrix} GM & IBM & AAPL \\ 700 & 400 & 200 \end{bmatrix}$$

At the close of trading on a certain day, the prices (in dollars per share) of these stocks are

$$B = \begin{bmatrix} GM & 27 \\ IBM & 205 \\ AAPL & 420 \end{bmatrix}$$

What is the total value of Judy's holdings as of that day?

Solution : Judy's holdings are worth

$$AB = \begin{bmatrix} 700 & 400 & 200 \end{bmatrix}\begin{bmatrix} 27 \\ 205 \\ 420 \end{bmatrix} = (700)(27) + (400)(205) + (200)(420) = 5184,900$$

or $\$184,900$.

14. **Production Planning :** Ace Novelty received an order from Magic World Amusement Park for 900 Giant Pandas, 1200 Saint Bernards, and 2000 Big Birds. Ace's management decided that 500 Giant Pandas, 800 Saint Bernards, and 1300 Big Birds could be manufactured in their Los Angeles plant, and the balance of the order could be filled by their Seattle plant. Each Panda requires 1.5 square yards of plush, 30 cubic feet of stuffing, and 5 pieces of trim; each Saint Bernard requires 2 square yards of plush, 35 cubic feet of stuffing, and 8 pieces of trim; and each Big Bird requires 2.5 square yards of plush, 25 cubic feet of stuffing, and 15 pieces of trim. The plush costs \$4.50 per square yard, the stuffing costs 10 cents per cubic foot, and the trim costs 25 cents per unit.

(a) How much of each type of material must be purchased for each plant?

(b) What is the total cost of materials incurred by each plant and the total cost of materials incurred by Ace Novelty in filling the order?

Solution The quantities of each type of stuffed animal to be produced at each plant location may be expressed as a 2 × 3 *production matrix* P. Thus,

$$P = \begin{array}{l} \text{L.A.} \\ \text{Seattle} \end{array} \overset{\begin{array}{ccc} \text{Pandas} & \text{St. Bernards} & \text{Birds} \end{array}}{\begin{bmatrix} 500 & 800 & 1300 \\ 400 & 400 & 700 \end{bmatrix}}$$

Similarly, we may represent the amount and type of material required to manufacture each type of animal by a 3 × 3 *activity matrix* A. Thus,

$$A = \begin{array}{l} \text{Pandas} \\ \text{St. Bernards} \\ \text{Birds} \end{array} \overset{\begin{array}{ccc} \text{Plush} & \text{Stuffing} & \text{Trim} \end{array}}{\begin{bmatrix} 1.5 & 30 & 5 \\ 2 & 35 & 8 \\ 2.5 & 25 & 15 \end{bmatrix}}$$

Finally, the unit cost for each type of material may be represented by the 3 × 1 *cost matrix* C.

$$C = \begin{array}{l} \text{Plush} \\ \text{Stuffing} \\ \text{Trim} \end{array} \begin{bmatrix} 4.50 \\ 0.10 \\ 0.25 \end{bmatrix}$$

a. The amount of each type of material required for each plant is given by the matrix PA. Thus,

$$PA = \begin{bmatrix} 500 & 800 & 1300 \\ 400 & 400 & 700 \end{bmatrix} \begin{bmatrix} 1.5 & 30 & 5 \\ 2 & 35 & 8 \\ 2.5 & 25 & 15 \end{bmatrix}$$

$$= \begin{array}{l} \text{L.A.} \\ \text{Seattle} \end{array} \overset{\begin{array}{ccc} \text{Plush} & \text{Stuffing} & \text{Trim} \end{array}}{\begin{bmatrix} 5600 & 75{,}500 & 28{,}400 \\ 3150 & 43{,}500 & 15{,}700 \end{bmatrix}}$$

b. The total cost of materials for each plant is given by the matrix PAC:

$$PAC = \begin{bmatrix} 5600 & 75{,}500 & 28{,}400 \\ 3150 & 43{,}500 & 15{,}700 \end{bmatrix} \begin{bmatrix} 4.50 \\ 0.10 \\ 0.25 \end{bmatrix}$$

$$= \begin{array}{l} \text{L.A.} \\ \text{Seattle} \end{array} \begin{bmatrix} 39{,}850 \\ 22{,}450 \end{bmatrix}$$

or \$39,850 for the L.A. plant and \$22,450 for the Seattle plant. Thus, the total cost of materials incurred by Ace Novelty is \$62,300.

Definition 1. *leontief input-output Model In a Leontief input-output model, the matrix equation giving the net output of goods and services needed to satisfy consumer demand is*

Total output		Internal consumption		Consumer demand
X	$-$	AX	$=$	D

where X is the total output matrix, A is the input-output matrix, and D is the matrix representing consumer demand.
The solution to this equation is

$$X = (I - A)^{-1} D \text{ Assuming that } (I - A)^{-1} \text{ exists.}$$

which gives the amount of goods and services that must be produced to satisfy consumer demand.

15. An input-output Model for a Three-Product Company TKK Corporation, a large conglomerate, has three subsidiaries engaged in producing raw rubber, manufacturing tires, and manufacturing other rubber-based goods. The production of 1 unit of raw rubber requires the consumption of 0.08 unit of rubber, 0.04 unit of tires, and 0.02 unit of other rubber-based goods. To produce 1 unit of tires requires 0.6 unit of raw rubber, 0.02 unit of tires, and 0 unit of other rubber-based goods. To produce 1 unit of other rubber-based goods requires 0.3 unit of raw rubber, 0.01 unit of tires, and 0.06 unit of other rubber-based goods. Market research indicates that the demand for the following year will be \$200 million for raw rubber, \$800 million for tires, and \$120 million for other rubber-based products. Find the level of production for each subsidiary in order to satisfy this demand.
Solution: View the corporation as an economy having three sectors and with an input-output matrix given by

$$A = \begin{array}{l} \text{Raw rubber} \\ \text{Tires} \\ \text{Goods} \end{array} \overset{\begin{array}{ccc} \text{Raw rubber} & \text{Tires} & \text{Goods} \end{array}}{\begin{bmatrix} 0.08 & 0.60 & 0.30 \\ 0.04 & 0.02 & 0.01 \\ 0.02 & 0 & 0.06 \end{bmatrix}}$$

Using Equation above, we find that the required level of production is given by

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = (I - A)^{-1} D$$

where x, y, and z denote the outputs of raw rubber, tires, and other rubber-based goods and where

$$D = \begin{bmatrix} 200 \\ 800 \\ 120 \end{bmatrix}$$

Now,

$$I - A = \begin{bmatrix} 0.92 & -0.60 & 20.30 \\ -0.04 & 0.98 & -0.01 \\ -0.02 & 0 & 0.94 \end{bmatrix}$$

You are asked to verify that

$$(I - A)^{-1} = \begin{bmatrix} 1.12 & 0.69 & 0.37 \\ 0.05 & 1.05 & 0.03 \\ 0.02 & 0.01 & 1.07 \end{bmatrix}$$

Therefore,

$$(I - A)^{-1} D = \begin{bmatrix} 1.12 & 0.69 & 0.37 \\ 0.05 & 1.05 & 0.03 \\ 0.02 & 0.01 & 1.07 \end{bmatrix} \begin{bmatrix} 200 \\ 800 \\ 120 \end{bmatrix} = \begin{bmatrix} 820.4 \\ 853.6 \\ 140.4 \end{bmatrix}$$

To fulfill the predicted demand, \$820 million worth of raw rubber, \$854 million worth of tires, and \$140 million worth of other rubber-based goods should be produced.