Numerical Method **(Math-2073)**
Chapter 6-7 problem

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" Problems cannot be solved at the same level of awareness that created them." Albert Einstein.

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***Note**: it is the 1.1 version, if you have any comment and suggestion please contact me.*

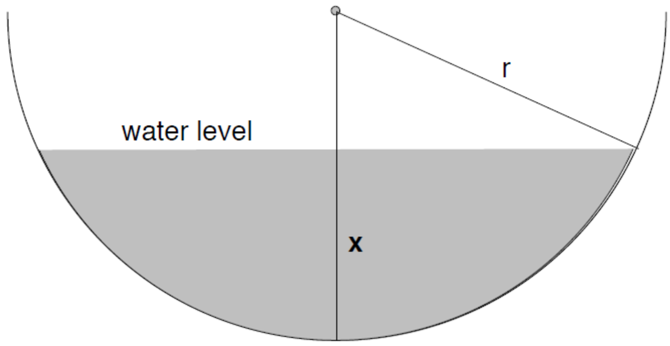
1. Water is flowing from a reservoir shaped like a hemisphere bowl of radius 12 m (see figure below). The volume of water is given by

$$V(x) = \frac{\pi}{3}x^3(3r - x).$$

The rate of change of the volume of water is

$$dV/dt = (dV/dx)(dx/dt).$$

Assume that $dx/dt = 1$ and approximate $dV/dt = dV/dx$ when $x = 3$ using the central-difference formula with $h = 0.1, 0.01$, and 0.001.



2. Using the composite trapezoidal rule and the composite Simpson rule(1/3 and 3/8) find $\int_a^b f(x)dx$. Up to fourth iteration
   (a) $f(x) = x^3 + x^2 - x + 1/x - 1/x^2,\ a = -3, b = -1$
   (b) $f(x) = \sin x + \cos x + e^x, a = -1, b = 1$

3. Compare numerical results for $\int_1^2 \ln x dx$ achieved when using the simple trapezoidal rule and the simple Simpson's rule to the analytically result. Explain any differences or similarities.

4. With precision $\xi = 0.0001$ apply the composite trapezoidal rule on

$$\int_0^{\frac{\pi}{4}} \sin(\frac{x}{8})dx.$$

5. Using a method of your choice calculate

$$\int_{-0.5}^{0.7} 2\sin 3x dx.$$

Any result which differs less than 20% from the exact value of the integral will be accepted.

6. Evaluate the integral $\int_0^3 \frac{1}{e^{-x^2}}dx$ with step length h=0.5 by using
   (a) Trapezoidal rule
   (b) Simpson's 1/3 and 3/8 rule

7. To monitor the thermal pollution of a river, a biologist takes hourly temperature T reading (in ^oF) from $9AM$ to $4PM$. The results are shown in the following table.

Time of day	9	10	11	12	13	14	15	16
Temperature	75.3	77.0	83.2	84.8	86.5	86.4	81.1	78.6

   Use Simpson's rule to estimate the average water temperature between $9AM$ and $4PM$ given by

$$T_{av} = \frac{1}{b-a}\int_a^b T(t)dt$$

8. Assume that Kulfo river is $80m$ wide. The depth of the river at a distance of x from the Campus is given by the following data

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

   Find the approximate area of cross section of Kulfo river.

9. The table below reveals the velocity v of athlete Haile G/Slasie during the time t.

t	1	1.1	1.2	1.3	1.4
v	43.1	47.5	51.3	57.1	63.5

   (a) Find the acceleration of Hiale at $t = 1.2$.
   (b) Find the total distance covered by Hiale from $t = 1$ to $t = 1.4$

10. Find the particular solution of the given ordinary differential equations on the respective intervals $I = [a, b]$ with step h. Find the solution using the Euler method, it's modifications and using the classical Runge-Kutta method.
   (a) $y' = x^2 - y, y(0) = 1,\ I = [0, 1], h = 0.2$
   (b) $y' - y - x = \sin(x + y);\ y(1) = 2$. With step h = 0.2 find y(1.4).
   (c)

11. Suppose water is leaking from a tank through a circular hole of area Ah at its bottom. Friction and contraction of a water stream near the hole reduce the volume of water leaving the tank per second to $cA_h\sqrt{2gh}$, where $0 < c < 1$ is a constant. The differential equation for the height h of water at time t for a cubical tank with a side $10ft$ and a hole of radius $2ft$ is

$$\frac{dh}{dt} = -\frac{c\pi}{450}\sqrt{h}.$$

Suppose the tank is initially full and $c = 0.4$, find the height of water after 3 seconds using the Runge-Kutta method of order 2.

12. In the study of nonisothermal flow of Newtonian fluid between parallel plates, the initial-value problem of the form

$$y'' = -t^2 e^y, \qquad y(0) = 1, y'(0) = 1$$

arises. Use the Runge-Kutta method of order 4 to approximate $y(1)$ with $h = 0.05$

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