Problem	4
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Target group:	Second year civil and production A
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Solve the flowing problems manually (if possible) and by using computer Lab.
Note: $*$ for project problems

1 Interpolation

1. **(Conceptual)** Answer True or False and give reasons for your answer.
   (a) Different methods of interpolation give different interpolating polynomials.
   (b) The condition numbers of the linear systems arising from interpolation with different bases are the same.
   (c) The interpolation error is the same no matter what method of interpolation is used.
   (d) The polynomial of degree $\leq n$ which interpolates $f(x)$ at $(n+1)$ distinct points is $f(x)$, if $f(x)$ itself is a polynomial of degree $\leq n$.
2. Based on your observation, is the interpolating polynomial always of degree N for $N+1$ data points? Explain why.
3. For the data $(-2,0),(0,4),(1,9)$.
   (a) Determine power series form of the interpolating polynomial.
   (b) Determine Lagrange form of the interpolating polynomial.
   (c) Construct a divided difference table. Then determine the the Newton divided difference interpolating polynomial.
   (d) Do your answer for (a) (b) and (c) agree? Explain why.

4. Given the data

x	0	1	2	4	6
f	1	9	23	93	259

do the following.

   (a) Construct the divided-difference table.

   (b) Using Newton's interpolation polynomial, find an approximation to $f(4.2)$.

5. Write code to compute the coefficients in the Newton divided difference interpolating polynomial from the divided difference table.

6. From the table, Estimate the number of students who obtained marks between 40 and 45.

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

7. The following table give the marks secured by 100 students in Numerical Methods:

Range of marks	30-40	40-50	50-60	60-70	70-80
No. of students	25	35	22	11	7

   Use Newton's forward difference interpolation formula to find

   (a) the number of students who got more than 55 marks.

   (b) the number of students who secured marks in the range from 36 to 45.

8. The percentage of Criminals for different age group are given below:

Age less than :	25	30	40	50
Percentage of Criminals :	52	67	84	94

   Apply Lagrange's formula to find the percentage of criminals under 35 years of age.

9. Write a code for interpolation polynomials

   (a) Use your code to determine the polynomial interpolating the following data:

x	1	2	3	-4	5
y	2	48	272	1182	2262

(b) Compute the value of the interpolating polynomial when $x = -1$.

10. Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0), (0.5, y), (1, 3)$ and $(2, 2)$. Find y if the coefficient of x^3 in $P_3(x)$ is 6.

11. For a function f, the forward divided differences are given by

$$\begin{array}{rcll} x_0 & = & 0.0 & f[x_0] \\ & & & \qquad f[x_0, x_1] \\ x_1 & = & 0.4 & f[x_1] \qquad\qquad f[x_0, x_1, x_2] = \dfrac{50}{7} \\ & & & \qquad f[x_1, x_2] = 10 \\ x_2 & = & 0.7 & f[x_2] = 6 \end{array}$$

Determine the missing entries.

12. For the following sets of points $[x_i, f_i]$ decide whether the polynomial $P(x)$ is their interpolation polynomial.

(a) $P_3(x) = x^3 + 2x^2 - 4x + 1$ for points $[-1, 6], [2; 9], [3, 34]$

(b) $P_3(x) = x^3 + 2x^2 - 4x + 1$ for points $[-2, 9], [-1, 6], [2, 9], [3, 34]$

(c) $P_3(x) = x^3 + 2x^2 - 4x + 1$ for points $[-2, 9], [-1, 6], [2, 9], [3, 34], [5, 130]$

(d) $P_3(x) = x^3 + 2x^2 - 4x + 1$ for points $[-2, 9], [-1, 6], [2, 9], [3, 34], [5, 156]$

(e) $P_1(x) = x + 1$ for points $[2, 3], [3, 4], [5, 6], [7, 8]$

13. The vapor pressure P of water (in bars) as a function of temperature $T(^oC)$ is

T	0	10	20	30	40	60	80	100
P	0.0061	0.0123	0.0234	0.0424	0.0738	0.1992	0.4736	1.0133

Find the interpolating polynomial of these data and estimate $P(5), P(45)$, and $P(95)$. Compare your results with the known values of the pressure: $P(5) = 0.008721, P(45) = 0.095848, P(95) = 0.84528$.

14. Find the values of a, b, c, d, e, and f such that the following function defines a cubic spline and find $g(0), g(2)$, and $g(3)$.

$$g(x) = \begin{cases} 2x^3 + 4x^2 - 7x + 5, & 0 \le x \le 1 \\ 3(x-1)^3 + a(x-1)^2 + b(x-1) + c, & 1 \le x \le 2 \\ (x-2)^3 + d(x-2)^2 + e(x-2) + f, & 2 \le x \le 3 \end{cases}$$

15. A biologist who is studying the growth of a bacteria culture recorded the following data

t	0	3	6	9
p(t)	2.0	3	4	6

where $p(t)$ denotes the number of bacteria at time t (minutes). Use a natural cubic spline to estimate the number of bacteria at $t = 5$.

2 Least square

1. The following table lists the total water usage in the United States in billions of gallons per day.

Year	1930	1940	1950	1960	1970
Water Use	110.2	137.4	202.6	322.7	411.1

   (a) Find the least-squares exponential of the water consumption on time.

   (b) Use the results of part (a) to predict the water consumption in 1980 and 1990.

2. A small company has been in business for 3 years and has recorded annual profits (in thousands of dollars) as follows: 4 in the first year, 2 in the second year, and 1 in the third. Find a linear function that approximates the profits as a function of the years, in the least-squares sense.

3. The following table lists the number of motor vehicle accidents in the United States for various years from 1950 to 1968.

Year	1950	1955	1960	1965	1966	1967	1968
No. Accidents (in thousands)	8,200	9,800	10,300	13,100	13,500	13,600	14,500
Accidents per 10^4 Vehicles	1,687	1,576	1,396	1,438	1,418	1,384	1,414

(a) Find the linear least-squares of the number of accidents on time. Use it to predict the number of accidents in 1990.

(b) Compute the quadratic least-squares of the number of accidents per $10,000$ vehicles on time. Use it to predict the number of accidents per $10,000$ vehicles in 1990.

4. A biologist is doing an experiment on the growth of a certain bacteria culture. After 4 hours the following data has been recorded:

t	0	1	2	3	4
p	1.0	1.8	3.3	6.0	11.0

where t is the number of hours and p the population in thousands. Determine the least-squares exponential that best fits these data. Use your results to predict the population of bacteria after 5 hours.

5. The values of the concentration C of a desired compound in terms of the time t (sec.) are given in the following table

t	3	9	12	18	24 3	0
C	4.1	4.3	3.9	3.4	3.1	2.7

Assuming that the guess function is $C(t) = c + ae^{-0.47t} + be^{-0.06t}$, find the values of a, b, and c that best fit this table

6. The population p of a small city during the period $[1960, 2000]$ is given by the table

t	1960	1970	1980	1990	2000
p	12600	14000	16100	19100	23200

Use the least-squares quadratic to predict the population in the year 2010.

7. The crop height measurement from lab experiment is registered as a table blow. From this tabular result we need to estimate the parametric value of Gomez model

$$g(x) = k_e^{-k_2 e^{-k_3 x}}$$

X	10	15	30	45	60	75
Y	70	73	80	86	88	90

8. By using the same data in the above example we need to estimate the parameters of logistic growth model which is similar to saturation function

$$p(x) = k_1 + \frac{k_2}{1 + e^{-k_3 + k_4 x}}$$

9. Find the linear curve fit of the above non-linear Gomez and Logistic model by using the given data sets.