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Arba Minch University

Problem three

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Solve the flowing problems manually (if possible) and by using computer Lab.

1 Direct Method

1. Solve the following linear system by using Gauss elimination, cramer's rule and inverse method.

   (a) Solve the following equation system using Gaussian Elimination.

   $$\begin{aligned} x_1 + 2x_2 - x_3 - x_4 + x_5 &= 0 \\ x_1 + 2x_2 - 2x_4 + 4x_5 &= 0 \\ 2x_1 + 4x_2 - 2x_3 - 2x_4 + 2x_5 &= 0 \\ -2x_1 - 4x_2 + 4x_3 + 4x_5 &= 0 \end{aligned}$$

   (b) Determine the polynomial $p(x) = a_0 + a_1x + a_2x^2$ whose graph passes through the points (1, 4), (2, 0), and (3, 12).

   (c) Set up a system of linear equations to represent the network shown in Figure 1. Then solve the system.

2. Consider

   $$A = \begin{bmatrix} 1 & 6 & 2 \\ 2 & 12 & 5 \\ -1 & -3 & -1 \end{bmatrix}$$

   (a) Show that A does not have an LU decomposition.

   (b) Re-order the rows of A and find an LU decomposition of the new matrix.

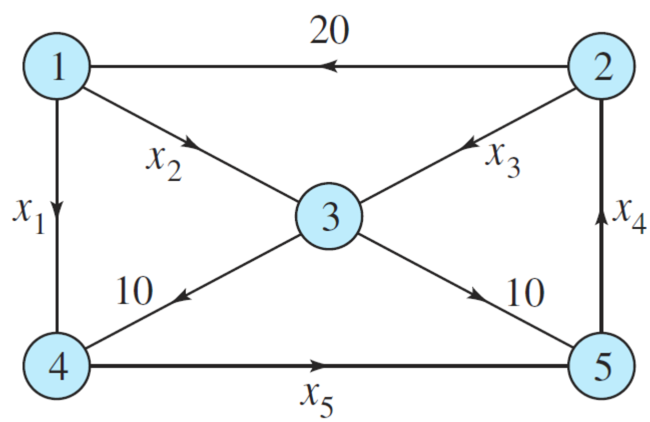


Figure 1:

(c) Hence solve

$$
\begin{aligned}
x_1 + 6x_2 + 2x_3 &= 9 \\
2x_1 + 12x_2 + 5x_3 &= -4 \\
-x_1 - 3x_2 - x_3 &= 17
\end{aligned}
$$

3. Consider the linear system

$$
\begin{pmatrix} 2 & 1 & 1 & 3 & 2 \\ 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 9 & 1 & 5 \\ 3 & 1 & 1 & 7 & 1 \\ 2 & 1 & 5 & 1 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 3 \\ -5 \\ 1 \end{pmatrix}
$$

(a) Solve the linear system AX = B by using the Doolittle method.

(b) Solve the linear system AX = B by using the Crout method.

(c) Solve the linear system AX = B by using the Cholesky method.

4. Suppose a university is comprised of three colleges: Sciences, Engineering, and Computer Science. The annual budgets for these colleges are \$16 million, \$5 million, and \$8 million, respectively. The full-time enrollments are 4000 in Sciences, 1000 in Engineering, and 2000 in Computer Science. The enrollment consists of 70% from Sciences, 20% from Engineering, and 10% from Computer

Science. For example, Engineering and Computer Science courses contain some students from other colleges. The distribution is as follows

	Courses taught by		
Students from	Sciences	Engineering	Computer Science
Sciences	70%	10%	15%
Engineering	20%	90%	10%
Computer Science	10%	0%	75%

Determine to the nearest cent the annual cost of educating a student in each of the three colleges.

5. Use both naive Gaussian elimination and Gaussian elimination with scaled partial pivoting to solve the following linear system using four-decimal floating point arithmetic

$$0.0003x_1 + 1.354x_2 = 1.357.$$
$$0.2322x_1 - 1.544x_2 = 0.7780.$$

6. Use Gaussian elimination with scaled partial pivoting and two-digit rounding arithmetic to solve the linear system

$$0.0001x + 1 + 66.x_2 = 65.$$
$$2.0x_1 - 28.x_2 = 33.$$

7. Consider the system
$$\begin{bmatrix} 2.1 & 1.8 \\ 6.2 & 5.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 6.2 \end{bmatrix}.$$
Solve the system using Gaussian elimination. Change the entry $a_{21} = 6.2$ to 6.1 and solve the system again. From the solutions obtained, what conclusion can be made about the system of equations?

2 Iterative Method

8. In the following Exercises perform 2 steps of the Jacobi's method. As a new exercise always perform 2 steps of the Gauss-Seidel method. In both cases choose $x^{(0)} = (0, 0, 0)$ for the initial approximation. Then compare your results

and the exact solution obtained using either Cramer's rule or the Gaussian elimination method. Prior to starting the iteration algorithms always check convergence of the chosen method. If convergence is not secured by the form of the linear system, try to secure it.

Results of all exercises are always included as decimal numbers rounded to four digits after the decimal point.

(a)

$$\begin{aligned} x_1 + 6x_2 - 8x_3 &= 6.6 \\ 10x_1 - 3x_2 + 6x_3 &= 8.6 \\ x_1 + 15x_2 - 9x_3 &= 0.8 \end{aligned}$$

(b)

$$\begin{aligned} 3x_1 + 10x_2 - 4x_3 &= 9 \\ 20x_1 + 2x_2 + 3x_3 &= 25 \\ 2x_1 - x_2 - +5x_3 &= 6 \end{aligned}$$

9. Consider the system of equation $Ax = b$ where

$$A = \begin{bmatrix} 1 & k \\ 2k & 1 \end{bmatrix}, k\, textreal$$

   (a) Find the values of k for which the matrix A is strictly diagonally dominant.

   (b) For $k = 0.25$, solve the system using the Jacobi method.

3 Power Method

1. The three-dimensional state of stress at a point is given by the stress tensor:

$$\sigma_{ij} = \begin{bmatrix} 40 & 20 & -18 \\ 20 & 28 & 12 \\ -18 & 12 & 14 \end{bmatrix} ksi$$

The principal stresses and the principal directions at the point are given by the eigenvalues and the eigenvectors.Use the power method for determining the value of the largest principal stress. Start with a column vector of ls, and carry out the first three iterations.

4 Non-linear system

1. Solve using modified Newton's method the following system of non-linear algebraic equations

$$
\begin{aligned}
x_1^3 - 2x_2 - 2 &= 0 \\
x_1^3 - 5x_2^2 + 7 &= 0 \\
x_2 x_3^2 - 1 &= 0
\end{aligned}
$$

2. Apply Newton's method to this problem with $x^0 = (0.1, 0.1, -0.1)$

$$
\begin{aligned}
3x_1 - \cos(x_2 x_3) - \frac{1}{2} &= 0 \\
x_1^2 - 81(x_2 + 0.1)^2 + \sin(x_3) + 1.06 &= 0 \\
e^{-x_1 x_2} + 20x_3 + \frac{10\pi - 3}{3} &= 0
\end{aligned}
$$

3. Solve the following nonlinear systems using Newton's method

   (a)

$$
\begin{aligned}
f_1(x, y) &= 3x - y - 3 = 0, \\
f_2(x, y) &= x - y + 2 = 0.
\end{aligned}
$$

   (b)

$$
\begin{aligned}
f_1(x, y) &= x^2 + 4y^2 - 16 = 0, \\
f_2(x, y) &= x^2 - 2x - y + 1 = 0.
\end{aligned}
$$

   (c)

$$
\begin{aligned}
f_1(x, y, z) &= x^2 + y^2 + z^2 - 9 = 0, \\
f_2(x, y, z) &= xyz - 1 = 0, \\
f_3(x, y, z) &= x + y - z^2 = 0.
\end{aligned}
$$

5 General

1. The upward velocity of a rocket is given at three different times

Time t(s)	Velocity v(m/s)
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as:

$$v(t) = a_1 t^2 + a_2 t + a_3, \qquad 5 \leq t \leq 12.$$

(a) Develop a system which describe this velocity in different time?

(b) Find the solution of the system by using (Do at least 5 iterations)

i. LU decomposition Method

ii. Jacobi method

iii. Gauss-Siedel Method

(c) Finding the absolute relative approximate error?

(d) Compare the result of above and discuss your suggestions?

2. Consider the loading of a statically determinate pin-jointed truss shown in Figure 2. The truss has seven members and five nodes, and is under the action of the forces R_1, R_2, and R_3 parallel to the y-axis.

Using the result that at each pin the sum of all forces F_i acting horizontally of vertically is equal to zero, find the member forces (F_i) obtained from the following system of equations.

$$\begin{bmatrix} 1.0 & -1.0 & 0 & 0.5 & -0.5 & 0 & 0 & \\ 0 & 0 & 0 & -8.66 & -8.66 & 0 & 0 & \\ 0 & 1.0 & 0 & 0 & 0 & 0.5 & 0 & \\ 0 & 0 & 0.5 & -0.5 & 0 & 0 & -1.0 & \\ 0 & 0 & 8.66 & 8.66 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0.5 & & -0.5 & 1.0 \\ 0 & 0 & 0 & 0 & 8.66 & 8.66 & 0 & \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 500 \\ 0 \\ 0 \\ 550 \\ 0 \\ 600 \end{bmatrix}$$

3. A coffee shop specializes in blending gourmet coffees. From type A, type B, and type C coffees, the owner wants to prepare a blend that will sell for \$8.50 for a 1-pound bag. The cost per pound of these coffees is \$12, \$9, and \$7, respectively. The amount of type B is to be twice the amount of type A. Write the system of equation that needs to be solved to find the amount of each type of coffee that will be in the final blend and solve the system using naive Gaussian elimination.

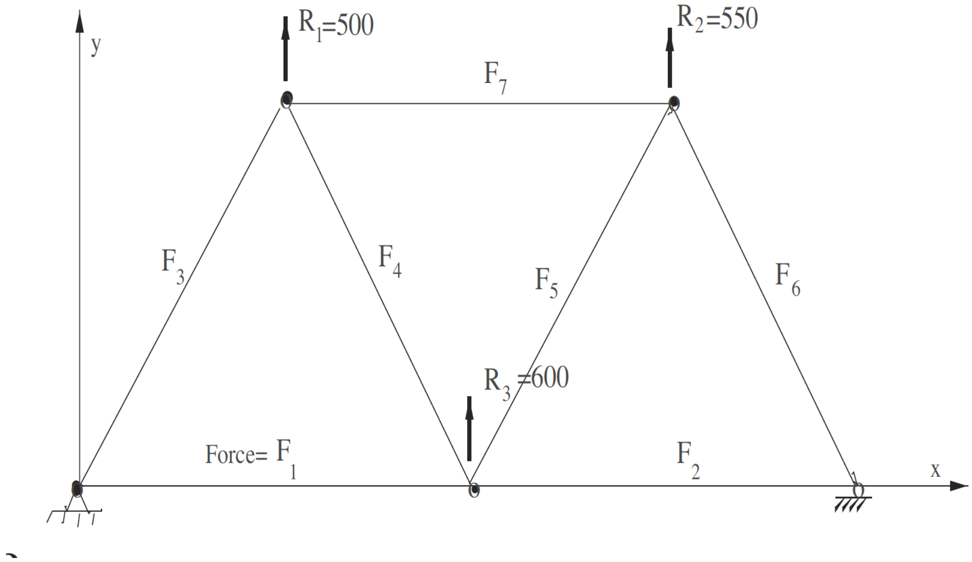
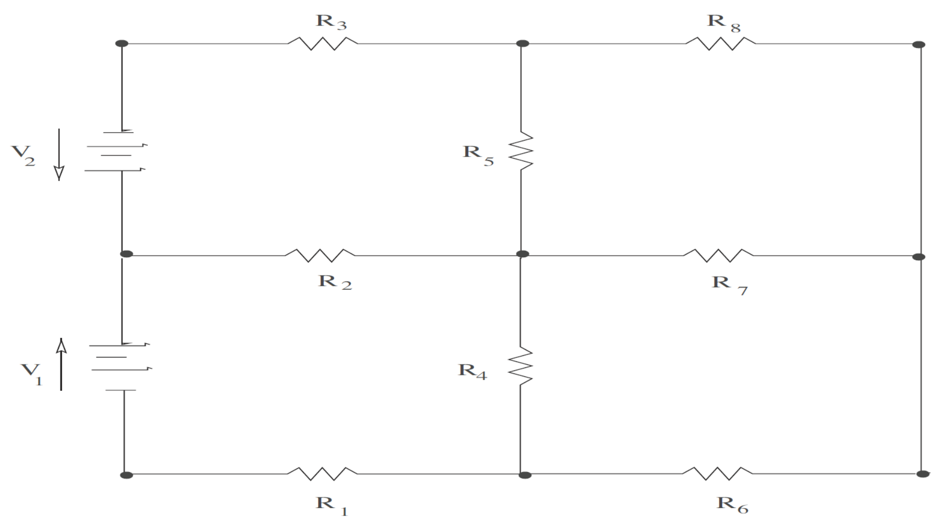


Figure 2: Determinate pin-jointed truss.

4. A company makes three types of patio furniture: chairs, rockers, and chaise lounges. Each requires wood, plastic, and aluminum, in the amounts shown in the following table. The company has in stock 400 units of wood, 600 units of plastic, and 1500 units of aluminum. For its end-of-the-season production run, the company wants to use up all the stock. To do this how many chairs, rockers, and chaise lounges should it make? (Hint: Let c be the number of chairs, r the number of rockers, and l the number of chaise lounges.)

	Wood	Plastic	Aluminum
Chair	1 unit	1 unit	2 units
Rocker	1 unit	1 unit	3 units
Chaise lounge	1 unit	2 units	5 units

5. A resistor network with two voltage sources is shown below. By applying both



Ohm's law and Kirchhoff's Current law, we get the following system of linear equations

$$\begin{bmatrix} R_1+R_2+R_4 & -R_2 & 0 & -R_4 \\ -R_2 & R_2+R_3+R_5 & -R_5 & 0 \\ 0 & -R_5 & R_5+R_7+R_8 & -R_7 \\ -R_4 & 0 & -R_7 & R_4+R_6+R_7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -V_1 \\ V_2 \\ 0 \\ 0 \end{bmatrix}.$$

Solve the system of equation if $R_1 = R_2 = R_3 = 100, R_4 = R_5 = 150, R_6 = R_7 = R_8 = 200, V_1 = 10$, and $V_2 = 12$. Note: The coefficient matrix is diagonally dominant.

6. Consider heat conduction in a small wire carrying electrical current that is producing heat at a constant rate. The equation describing the temperature $y(x)$ along the wire $(0 \le x \le 1cm)$ is

$$D\frac{\partial^2 y}{\partial x^2} = -S$$

with boundary conditions $y(0) = y(1) = 0^oC$, thermodiffusion coefficient $D = 0.01cm^2/sec$, and normalized source term $S = 1^oC/sec$. If we discretize the domain into 20 equal subintervals, using $x_j = j/20$ for $j = 0$ to 20, we can approximate the equation at x_j to obtain

$$D\frac{y_{j-1} - 2y_j + y_{j+1}}{h^2} = -S$$

where y_j is the temperature at $x = x_j$ and $h = 0.05$ is the step size. If we apply the boundary conditions at x_0 and x_20 we are left with 19 equations for 19 unknown temperatures, y_1 to y_19. We can put these equations into the matrix form $Ay = b$ where

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 & 0 \\ & & \vdots & & & & \\ 0 & 0 & \cdots & 0 & 1 & -2 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 & -2 \end{bmatrix}, y = \begin{bmatrix} 1 \\ y_2 \\ y_3 \\ \vdots \\ y_18 \\ y_19 \end{bmatrix}, b = \begin{bmatrix} -0.25 \\ -0.25 \\ -0.25 \\ \vdots \\ -0.25 \\ -0.25 \end{bmatrix}$$

Solve the above steady-state system problem by using the Jacobi iteration and Gauss-Seidel iteration. Start with $y = 0$.