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Arba Minch University

# Problem Two

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**Solve the flowing problems manually (if possible ) and by using computer Lab.**

## Bisection exercises

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1. Find the root of $f(x) = e^x - 2 - x$ in the interval $[-2.4, -1.6]$ accurate to $10^{-4}$ using the bisection method.

2. Use the bisection method to find solutions accurate to within $10^{-4}$ on the interval $[-5, 5]$ of the following functions:

   (a) $f(x) = x^5 - 10x^3 - 4$

   (b) $f(x) = 2x^2 + \ln(x + 6) - 3$

   (c) $f(x) = \ln(x + 1) + 30e^{-x} - 3$

3. The following equations have a root in the interval $[0, 1.6]$. Determine these with an error less than $10^{-4}$ using the bisection method.

   (a) $2x - e^{-x} = 0$

   (b) $e^{-3x} + 2x - 2 = 0$.

4. Estimate the number of iterations needed to achieve an approximation with accuracy $10^{-4}$ to the solution of $f(x) = x^3 + 4x^2 + 4x - 4$ lying in the interval $[0, 1]$ using the bisection method.

5. Use the bisection method for $f(x) = x^3 - 3x + 1$ in $[1, 3]$ to find:
   (a) The first eight approximation to the root of the given equation.
   (b) Find an error estimate $\mid \alpha - x_8 \mid$.

## False Position Exercises

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1. Solve the Problem 1 of bisection by the false position method.
2. Use the false position method to find the root of $f(x) = x^3 + 4x^2 + 4x - 4$ on the interval $[0, 1]$ accurate to $10^{-4}$.
3. Use the false position method to find solution accurate to within $10^{-4}$ on the interval $[1, 1.5]$ of the equation $2x^3 + 4x^2 - 2x - 5 = 0$.
4. Use the false position method to find solution accurate to within $10^{-4}$ on the interval $[3, 4]$ of the equation $e^x - 3x^2 = 0$.

## Fixed Point Iteration Exercises

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1. The cubic equation $x^3 - 3x - 20 = 0$ can be written as
   (a) $x = \frac{x^3-20}{3}$
   (b) $x = \frac{20}{x^3-3}$
   (c) $x = (3x + 20)^{1/3}$

   Choose the form which satisfies the condition $\mid g'(x) \mid < 1$ on $[1, 4]$ and then find third approximation $x_3$ when $x_0 = 1.5$.
2. Consider the nonlinear equation $g(x) = \dfrac{1}{2}e^{0.5x}$ defined on the interval $[0, 1]$. Then
   (a) Show that there exists a unique fixed-point for $g$ in $[0, 1]$.
   (b) Use the fixed-point iterative method to compute $x^3$, set $x_0 = 0$.

(c) Compute an error bound for your approximation in part (b).

3. An equation $x^3 - 2 = 0$ can be written in form $x = g(x)$ in two ways:

   (a) $x = g_1(x) = x^3 + x - 2$

   (b) $x = g_2(x) = \frac{2+5x-x^3}{5}$ Generate first four approximations from $x_{n+1} = g_i(x_n),\ i = 1, 2$ by using $x_0 = 1.2$.
   Show which sequence converges to $2^{1/3}$ and why ?

4. Find value of $k$ such that the iterative scheme $x_{n+1} = \frac{x_n^2 - 4kx_n + 7}{4}, n \geq 0$ converges to 1. Also, find the rate of convergence of the iterative scheme

5. Write the equation $x^2 - 6x + 5 = 0$ in the form $x = g(x)$, where $x \in [0, 2]$, so that the iteration $x_{n+1} = g(x_n)$ will converge to the root of the given equation for any initial approximation $x_0 \in [0, 2]$.

6. Which of the following iterations

   (a) $x_{n+1} = \frac{1}{4}(x_n^2 + \frac{6}{x_n})$

   (b) $x_{n+1} = (4 - \frac{6}{x_n})$

   is suitable to find a root of the equation $x^3 = 4x^2 - 6$ in the interval $[3, 4]$? Estimate the number of iterations required to achieve $10^{-3}$ accuracy, starting from $\mathrm{x}_0 = 3$.

7. An equation $e^x = 4x^2$ has a root in $[4, 5]$. Show that we cannot find that root using $x = g(x) = \frac{1}{2}e^{x/2}$ for the fixed-point iteration method. Can you find another iterative formula which will locate that root ? If yes, then find third iterations with $x_0 = 4.5$. Also find the error bound

## Newton Exercises

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1. Solve the Problem 1 of bisection' by the Newton's method by taking initial approximation $\mathrm{x}_0 = -2$.

2. Let $f(x) = e^x + 3x^2$

   (a) Find the Newton's formula $g(x_k)$.

(b) Start with $x_0 = 4$ and compute $x_4$.

(c) Start with $x_0 = -0.5$ and compute $x_4$.

3. Use the Newton's formula for the reciprocal of square root of a number 15 and then find the $3^{rd}$ approximation of number, with $x_0 = 0.05$.

4. Use the Newton's method to find solution accurate to within $10^{-4}$ of the equation $\tan(x) - 7x = 0$, with initial approximation $x_0 = 4$.

5. Find the Newton's formula for $f(x) = x^3 - 3x + 1$ in $[1, 3]$ to calculate $x_3$, if $x_0 = 1.5$. Also, find the rate of convergence of the method.

6. Find the root of the equation $f(x) = (x - 1)^3 + 0.512 = 0$. (**Divergence at inflection points**)

7. Find the root of the equation $f(x) = x^3 - 0.03x^2 + 2.4 \times 10^{-6} = 0$ at $x_0 = 0$, and $x_0 = 0.02$. **Division by Zero**

8. Find the root of the equation $f(x) = x^2 + 2 = 0$. **(Oscillations near local maximum and minimum)**

9. Find the root of the equation $f(x) = \sin(x) = 0$ at $x_0 = 2.4\pi$. **(Root Jumping)**

## Secant Exercises

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1. Find the positive root of $f(x) = x^{10} - 1$ by the secant method by using starting values $x_0 = 1.2$ and $x_1 = 1.1$ accurate to within $10^{-4}$.

2. Find the first three estimates for the equation $x^3 - 2x - 5 = 0$ by the secant method using $x_0 = 2$ and $x_1 = 3$.

3. Solve the equation $e^{-x} - x = 0$ by using the secant method, starting with $x_0 = 0$ and $x_1 = 1$, accurate to $10^4$.

4. Use the secant method to find a solution accurate to within $10^4$ for $\ln(x) + x - 5 = 0$ on $[3, 4]$.

# APPLIED PROBLEMS FOR CHAPTER 2

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1. The temperature in the interior of a material with imbedded heat sources is obtained from the solution of the equation

$$e^{-(1/2)t}\cosh^{-1}(e^{(1/2)t}) = \sqrt{k/2}.$$

Given that $k = 0.67$, find the temperature $t$.

2. A relationship for the compressibility factor $c$ of real gases has the form

$$\frac{1+x+x^2-x^3}{(1-x)^3}$$

If $c = 0.9$, find the value of $x$ in the interval $[-0.1, 0]$ using the bisection method.

3. One of the forms of the Colebrook Equation for calculating the Friction Factor $f$ is given by

$$1 + 2\log 10\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)\sqrt{f} = 0$$

where:

- $f$ is the Friction Factor and is dimensionless.
- $\epsilon$ is the Absolute Roughness and is in units of length.
- $D$ is the Inside Diameter and, as these formulas are written, is in the same units as $e$.
- $Re$ is the Reynolds Number and is dimensionless.
- $\epsilon/D$ is the Relative Roughness and is dimensionless.
- This equation can be solved for $f$ given the relative Roughness and the
- Reynolds number. Find $f$ for the following values of $\epsilon/D$ and $Re$.

(a) $\epsilon/D = 0.0001$ , $Re = 3 \times 10^5$.

(b) $\epsilon/D = 0.03, Re = 1 \times 10^4$.

(c) $\epsilon/D = 0.01, Re = 3 \times 10^5$..

4. Water is discharged from a reservoir through a long pipe (see Figure 3.13). By neglecting the change in the level of the reservoir, the transient velocity $v(t)$ of the water flowing from the pipe at time $t$ is given by

$$v(t) = \sqrt{2gh} \times \tanh\left(\frac{t}{2L}\sqrt{2gh}\right)$$

where $h$ is the height of the fluid in the reservoir, $L$ is the length of the pipe, $g = 9.81m/s^2$ is gravity. Find the value of $h$ necessary to achieve a velocity of $v = 4m/s$ at time $t = 4s$, when $L = 5m$. Use Newton's method for the calculation with the starting value $h_0 = 0.5$. The method should be stopped when the relative change in the solution is below $10^{-6}$.

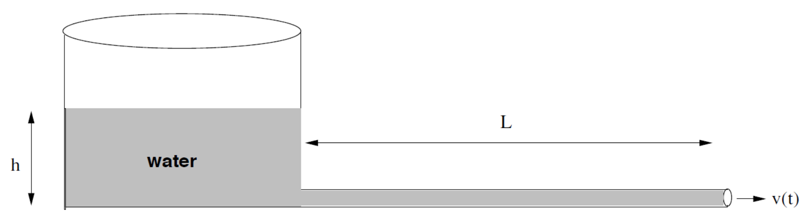


Figure 1: water discharge

## Department based problem

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Solve the problem by using all algorithms

1. You are making a bookshelf to carry books that range from 8 1/2" to 11" in height and would take up 29" of space along the length. The material is wood having a Young's Modulus of,3.667 MSI thickness of 3/8" and width of 12". You want to find the maximum vertical deflection of the bookshelf. The vertical deflection of the shelf is given by

$$v(x) = 0.42493 \times 10^{-4}x^3 - 0.13533 \times 10^{-8}x^8 - 0.66722 \times 10^{-6}x^4 - 0.018507x$$

where $x$ is the position along the length of the beam. Hence to find the maximum deflection we need to find where $f(x) = \frac{dv}{dx} = 0$ and conduct the second derivative test. The equation that gives the position x where the deflection is maximum is given by

$$-0.67665 \times 10^{-8}x^4 - 0.26689 \times 10^{-5}x^3 + 0.12748 \times 10^{-3}x^2 - 0.018507 = 0$$

Use the above method of finding roots of equations to find the position $x$ where the deflection is maximum. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration. **For civil only**

2. A trunnion has to be cooled before it is shrink fitted into a steel hub. The equation that gives the temperature $T_f$ to which the trunnion has to be cooled to obtain the desired contraction is given by

$$f(T_f) = -50598\times10^{-10}T_f^3+0.38292\times10^{-7}T_f^2+0.74363^{-4}T_f+0.88318\times10^{-2} = 0$$

Use the above method of finding root $T_f$ s of equations to find the temperature to which the trunnion has to be cooled. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration. **For production student**

Remark[1] Prepared by[2]

[1]This Homework problem is post only for one week

[2]Dejen Ketema `<dejen.ketema@amu.edu.et>`

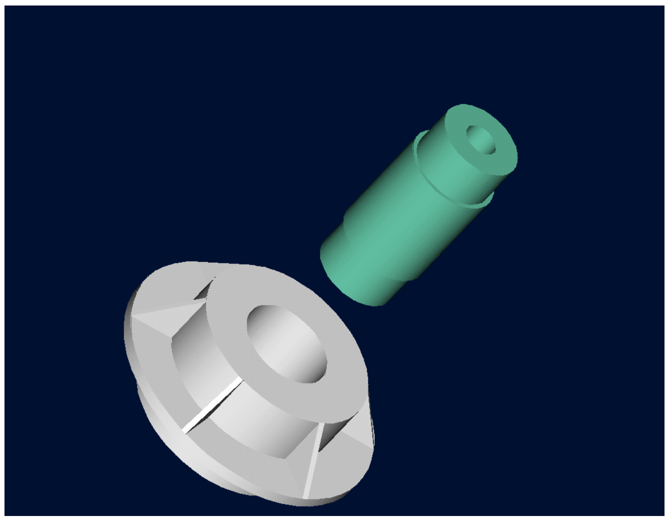

Figure 2: Trunnion to be slid through the hub after contracting