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Arba Minch University

Numerical Method(Math 2073/53)
Problem set 1

"Numerical methods" are methods devised to solve mathematical problems on a computer.

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1. Write down each of these numbers rounded them to **4 decimal places**:

$$0.12345,\ -0.44444,\ 0.5555555,\ 0.000127351,\ 0.000005$$

2. Write down each of these numbers, rounding them to **4 significant figures:**

$$0.12345,\ -0.44444,\ 0.5555555,\ 0.000127351,\ 25679$$

3. Show that the evaluation of the function

$$f(x) = x^2 - x - 1500$$

near $x = 39$ is an ill-conditioned problem.

4. Consider the function

$$f(x) = x^2 + x - 1975$$

and suppose we want to evaluate it for some x.

   (a) Let $x = 20$. Evaluate $f(x)$ and then evaluate f again having altered x by just 1%. What is the percentage change in f? Is the problem of evaluating $f(x)$, for $x = 20$, a well-conditioned one?

   (b) Let $x = 44$. Evaluate $f(x)$ and then evaluate f again having altered x by just 1%. What is the percentage change in f? Is the problem of evaluating $f(x)$, for $x = 44$, a well-conditioned one?

5. Perform the following calculations

Questions	a	b	c	d
Exact	17/15	4/15	139/660	301/660
3-digit chopping				
Relative error				
3-digit rounding				
Relative error				

6. Suppose that two points (x_0, y_0) and (x_1, y_1) are on a straight line with $y_1 \neq y_0$. Two formulas are available to compute the x-intercept of the line:

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} \quad \text{and} \quad x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0}.$$

   (a) Show that both formulas are algebraically correct.

   (b) Suppose that $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$. Use three-digit rounding arithmetic to compute the x-intercept using both of the formulas. Which method is better and why?

7. Compute $0.1 + 0.2 - 0.3$ in MATLAB

8. The distance from the Earth to the Moon varies between $356400km$ and $406700km$. Give a bound on the absolute and relative errors incurred when using one of both values as the "real distance." This requires giving two bounds

9. When computing $1 - \sin(\pi/2 + x)$ for small x, one commits a specific type of error. Which one? Can this be prevented?

10. Assume the Euro-peseta exchange value is $1Eu = 166.386birr$. However, Law requires that these transactions be rounded to the nearest monetary unit (i.e. either 1 birr or 1 cent). Compute

   (a) The absolute and relative errors incurred when exchanging 1 Euro for 166 birr.

   (b) The absolute and relative errors incurred when exchanging 1 birr for its "equivalent" in Euros.

   (c) The absolute and relative errors incurred when exchanging 1 cent for its "equivalent" in pesetas.

11. Consider the following two computations, the first one is assumed to be "correct" while the second one is an approximation:

$$26493 - \frac{33}{0.0012456} = -0.256 \ \text{(the exact result)}$$

.

$$26493 \frac{33}{0.0012456} = 8.2488 \ \text{(approximation)}$$

12. Consider $f(x) = \frac{1}{1+x}$

   (a) What is the second order Taylor polynomial of $f(x)$ around 0?

   (b) Use part (a) to approximate $f(0.1)$. What is the relative and absolute error of the approximation?

13. Convert the binary number to decimal format.

    (a) 1010100
    (b) 1101.001
    (c) 10110001110001.01010111

14. Converting decimal to binary

    (a) 157
    (b) 256.1875

15. Consider the function:

$$f(x) = x(\sqrt{x} - \sqrt{x-1})$$

    (a) Use MATLAB to calculate the value of $f(x)$ for the following three values of x: $x = 10, x = 1000, \quad \text{and } x = 100000$.
    (b) Use the decimal format with six significant digits to calculate $f(x)$ for the values of x in part(a). Compare the results with the values in part (a).
    (c) Change the form of $f(x)$ by multiplying it by $\dfrac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}$.Using the new form with numbers in decimal format with six significant digits, calculate the value of $f(x)$ for the three values of x. Compare the results with the values in part (a).

16. Find the rates of convergence of the following functions as $n \to \infty$:

    (a) $\lim_{n\to\infty} \sin(\frac{1}{n}) = 0$
    (b) $\lim_{n\to\infty} \sin(\frac{1}{n^2}) = 0$
    (c) $\lim_{n\to\infty} (\sin(\frac{1}{n}))^2 = 0$
    (d) $\lim_{n\to\infty} (\log(n+1) - \log(n))$
    (e) $\lim_{h\to 0} \frac{\sin h}{h}$