Numerical Method

Lecture-1

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What?

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Why?

Since the mid 20th century, the growth in power and availability of digital computers has led to an increasing use of realistic mathematical models in science and engineering, and numerical analysis of increasing sophistication is needed to solve these more detailed models of the world. The formal academic area of numerical method ranges from quite theoretical mathematical studies to computer science issues.

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OR

The overall goal of the field of numerical analysis is the design and analysis of techniques to give approximate but accurate solutions to hard problems,

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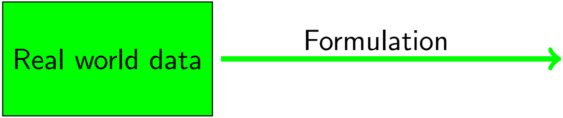
- Nowadays, mathematical modeling has a key role also in fields such as the environment and industry.
- One of the reasons for this growing success is definitely due to the impetuous progress of scientific computation;
- this discipline allows the translation of a mathematical model-which can be explicitly solved only occasionally-into algorithms that can be treated and solved by ever more powerful computers.

Modeling Process

Figure: Modeling Process

Modeling Process



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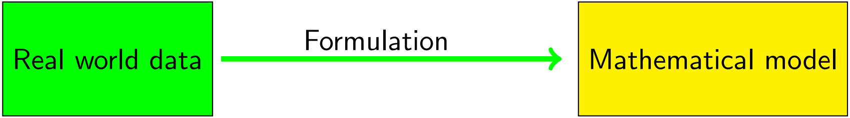

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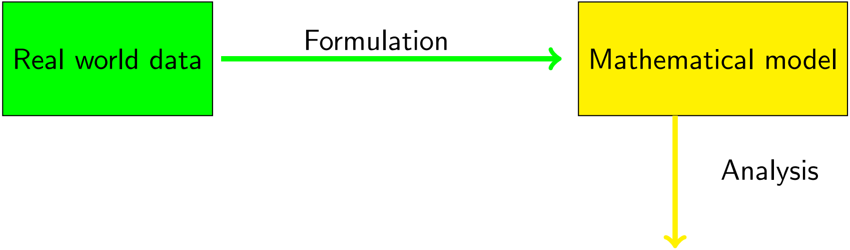

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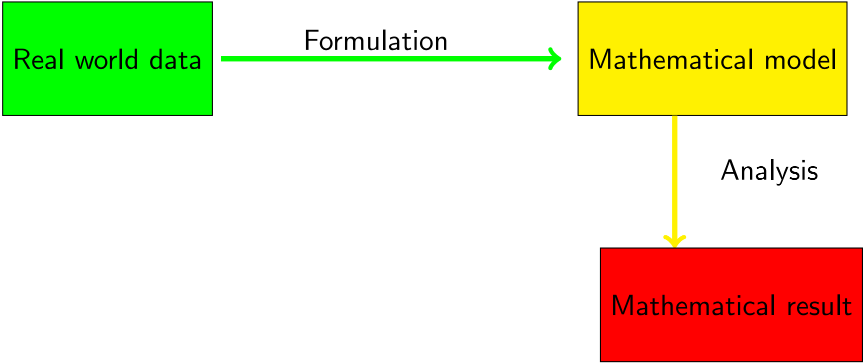

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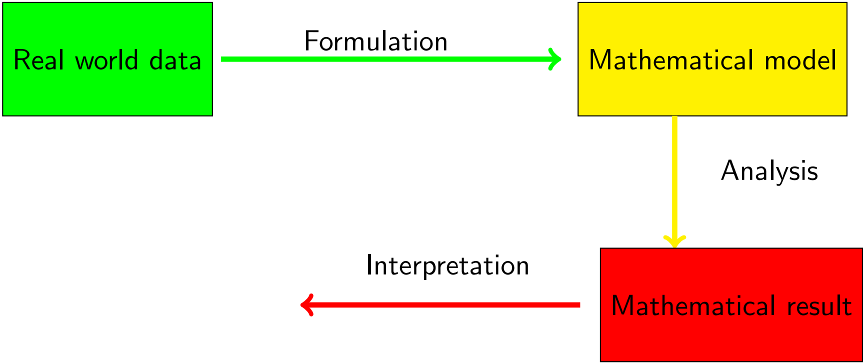

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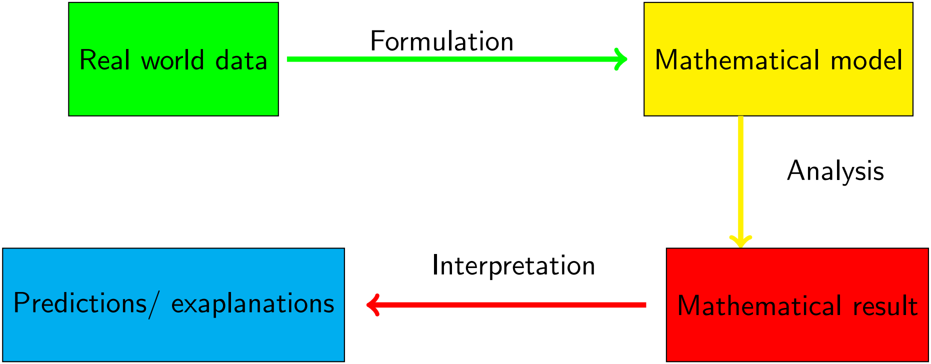

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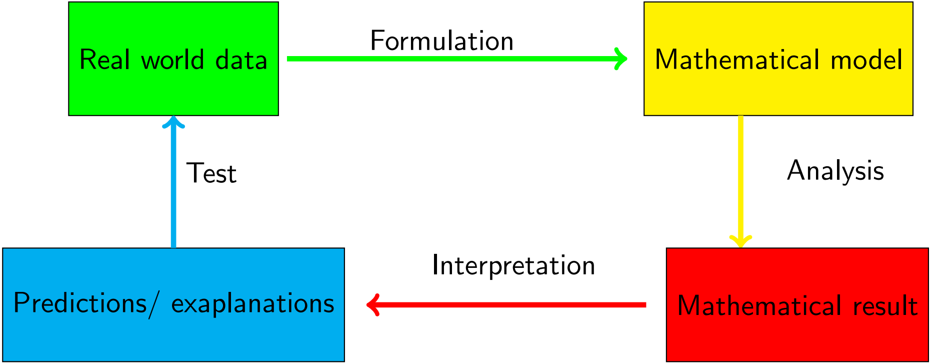

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$$V = L \times W \times D.$$

Definition

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- Mathematical model which has been formulated in an attempt to explain and understand an observed phenomenon in different discipline.
- We will concentrate on those mathematical models which are continuous (or piece-wise continuous) and are difficult or impossible to solve analytically:
- this is usually the case in practice.

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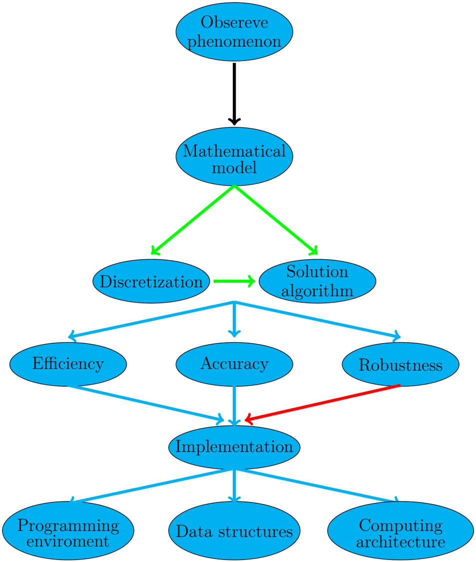
- While scientific computing focuses on the design and the implementation of such algorithms,
- This leads to questions involving programming languages, data structures, computing architectures and their exploitation (by suitable algorithms), etc.


Obsereve phenomenon
Mathematical model
Discretization
Solution algorithm
Efficiency
Accuracy
Robustness
Implementation
Programming enviroment
Data structures
Computing architecture

Representation of Numbers on a Computer

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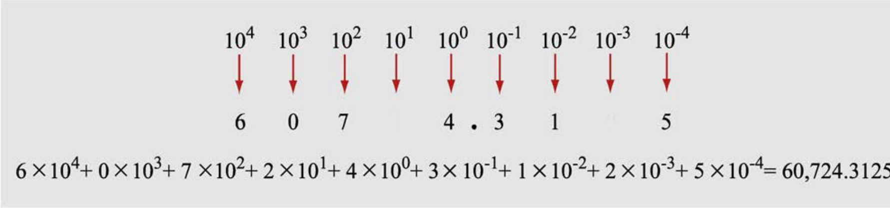

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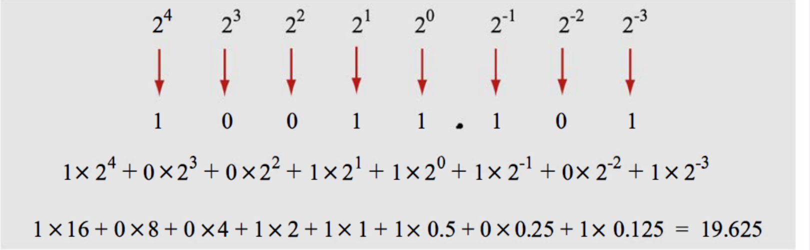

Binary Representation

The Binary Floating Point Arithmetic Standard 754-1985 (IEEE — The Institute for Electrical and Electronics Engineers) standard specified the following layout for a 64-bit real number:

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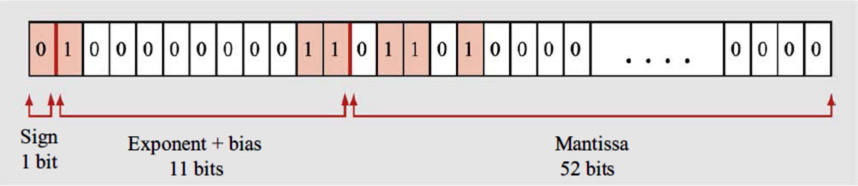

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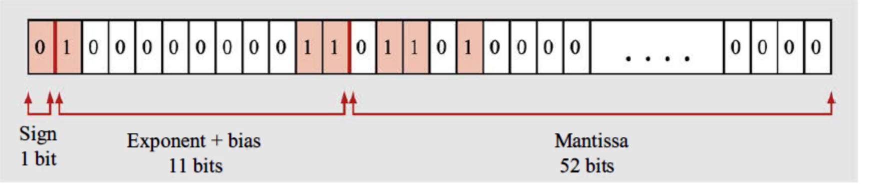

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- The largest positive number that can be expressed in double precision is approximately : $2^{1024} \approx 1.8 \times 10^{308}$.
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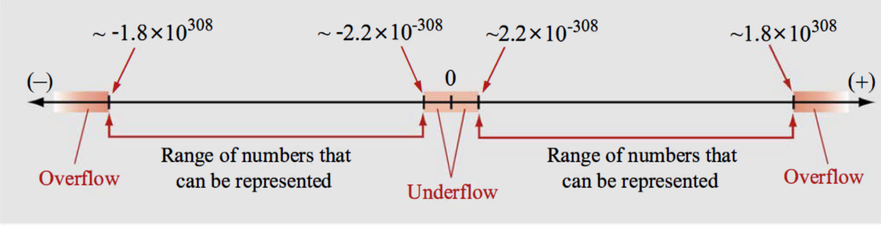
- 

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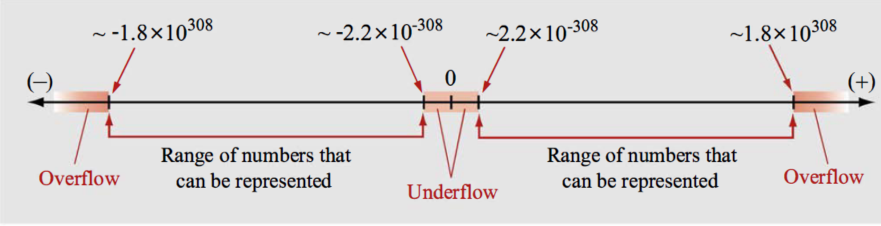
- 

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- This value is also defined as the machine epsilon in double precision.

Formula

$$r = (-1)^s 2^{c-1023}(1+m); \qquad c = \sum_{k=0}^{10} c_k 2^k, \qquad m = \sum_{k=0}^{51} \frac{m_k}{2^{52-k}}$$

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Example

Write the number 50 in binary floating point representation then change to decimal format(base 10)